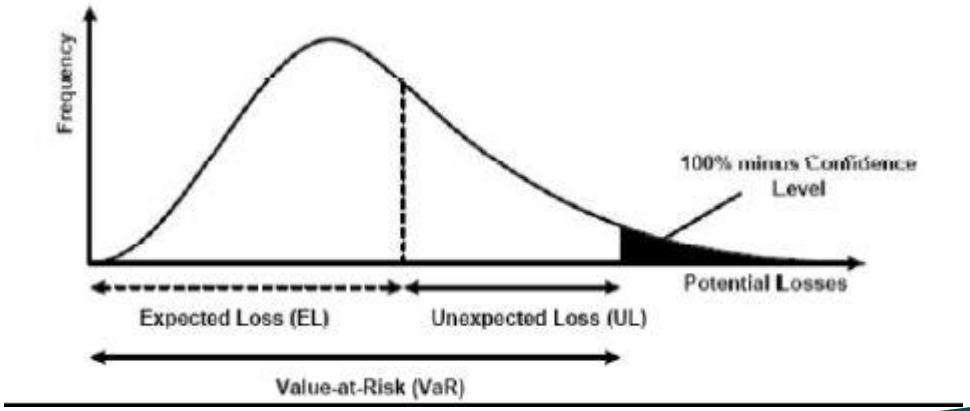
Historical Value at Risk

Historical VaR

Value at Risk Definition

- The maximum likely loss on a portfolio for a given probability defined as *x%* confidence level over N days
- Pr(*Loss* > *VaR* (*x%*)) < 1- *x%*



Value at Risk Roles

- Risk management
- Risk control
- Financial reporting
- Regulatory and economic capital

Value at Risk Pros & Cons

- Pros
  - Regulatory measurement for market risk
  - Objective assessment
  - Intuition and clear interpretation
  - Consistent and flexible measurement
- Cons
  - Doesn’t measure risk beyond the confidence level: tail risk
  - Non sub-additive

Historical VaR

Three Value at Risk Approaches

- Parametric Value at Risk
- Historical Value at Risk
- Monte Carlo Value at Risk

Historical Value at Risk

- Assumption

  The past is a good indicator of the near-future or history repeats itself

- Pros
  - Simple and intuitive
  - Easy back and stress test
  - No distribution assumption
  - No calibration
- Cons
  - Poor accuracy for higher confidence level and tail risk
  - Difficult for long horizons
  - Limited scenario

Historical Value at Risk Methodology and Implementation

- Obtain one year historical value time series of all market factors, such as a stock price time series is $\bar{x}_1 \quad \cdots \quad \bar{x}_{251}$
- Assuming today's value is x_0, generate 250 historical scenarios. The i-th is $x_i = (\bar{x}_i / \bar{x}_{i-1} - 1)x_0$
- Compute base PV at today t as $P(x_o)$
- Compute 250 scenario PVs: $P(x_i)$
- Compute 250 scenario P&L: $P(x_i) - P(x_0)$
- Sort 250 scenario P&L. The Value at Risk is the average between 2nd and 3rd lowest (negative) numbers

Value at Risk Scaling

- Normally firms compute 1-day 99% Value at Risk
- Regulators require 10-day 99% Value at Risk
- Under IID assumption, 10-day Value at Risk = $\sqrt{10} * VaR_{1-day}$

Value at Risk Backtest

- The only way to verify a Value at Risk system is to backtest
- At a certain day, compute hypothetic P&L. If (hypothetic P&L > VaR) ➔ breach, otherwise, ok
- Hypothetic P&L is computed by holding valuation date and portfolio unchanged
- In one year period,
  - If number of breaches is 0-4, the Value at Risk system is in Green zone
  - If number of breaches is 5-9, the Value at Risk system is in Yellow zone
  - If number of breaches is 10 or more, the Value at Risk system is in Red zone

Reference:

https://finpricing.com/lib/EqConvertible.html